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Comment on "Basin inversion and fault reactivation in laboratory experiments"

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Abstract

A recent paper describing laboratory experiments on fault reactivation misinterprets the principles of scale models of physical systems by using inconsistent dimensionless ratios that determine the correspondence between model and nature. For example the dimensionless stress ratio is not consistent with the dimensionless time ratio. Moreover, consistent, independent fundamental ratios of mass, length and time cannot be derived from the values of the variables and constants known in both the model and nature. Consequently, little information about the natural system can be derived from the model.

Keywords: Physical model; Analogue modelling

1. Introduction

Papers that attempt to model natural geological deformation sometimes misinterpret and misuse the principles of scaling and conclude more than is warranted. The purpose of this comment is to point out some of those misinterpretations in the recent paper by Del Ventisetti. et al. (2006).

During the early 1900s physicists were exploring the nature of dynamically similar systems, dimensionless ratios and scale invariant equations. In 1931, P.W. Bridgman's book on Dimensional Analysis summarized, refined and further developed the theory and explicitly stated the principle of dynamic similarity (Bridgman, 1931).

Dimensional analysis is based on the principle that any complete mathematical equation describing a physical process can be written as the product of the variables expressed as dimensionless ratios. If the variables involved in the physical process are known, then information about the nature of the functional relationship can be obtained.

In the first half of the last century dimensional analysis and scale modeling were important in understanding dynamically complex systems that could not be solved analytically. With the invention of digital computers and numerical methods for solving partial differential equations, dimensional analysis and scale modeling have lost much of their earlier importance.

In 1937, M.K. Hubbert published a paper on the theory of scale models using a different approach than Bridgman's. Hubbert's (1937) approach has dominated the thinking of structural geologists ever since. He began with a postulate for dynamic similarity claiming that the ratios of the various kinds of forces between the model and nature ($F^* = F_{\text{model}}/F_{\text{nature}}$), the masses ($M^* = M_{\text{model}}/M_{\text{nature}}$), and velocities ($V^* = V_{\text{model}}/V_{\text{nature}}$) all had to have the same values at corresponding points in the model and natural system. Given that starting point, the ratios of the fundamental units could be established from which all the other ratios between the model and nature, such as material properties, could be derived. The fundamental units are independent of each other and define the dimensions of all the variables that describe the system. In mechanical systems, there are only three fundamental

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Table 1
All units are SI; symbols with an * are dimensionless ratios of the parameter in the first column (measured model value, column 2, divided by the estimated natural value, column 3)

Parameter	Measured model value	Estimated natural value	Model/nature ratio	Symbol	M, L, T dimensions
Brittle layer density ($kg\,m^{-3)}$	1300	2600	0.5	ρ^*	M^*L^{*-3}
Brittle layer friction coefficient (dimensionless)	0.8	0.8	1.0	μ^*	None
Ductile layer density ($kg\,m^{-3}$)	1060	2200	0.481818182	ρ^*	M^*L^{*-3}
Ductile layer viscosity (Pa s)	1.00E + 03	5.00E + 17	2E − 15	η^*	$M^*L^{*-1}T^{*-1}$
Gravity ($m\,s^{-2}$)	9.81	9.81	1	g^*	L^*T^{*-2}
Length (m)	0.01	1000	1.00E − 05	l^*	L^*
Time (t, s)	3.60E + 03	1.30E + 13	2.77E − 10	t^*	T^*
Rate ($m\,s^{-1}$)	2.70E − 06	7.90E − 11	3.42E + 04	v^*	L^*/T^*
Strain rate (s^{-1})	6.55E − 04	1.93E − 13	3.40E + 09	ε^*	T^{*-1}

Lower case symbols in column 5 refer to the parameter in column 1. The upper case symbols in column 6 refer to the three fundamental units or dimensionless ratios of mass (M^*) length (L^*) and time (T^*) that characterize the parameter in column 1.Information in columns 1–4 comes from Table 2 and Appendix A in the paper by Del Ventisetti et al. (2006).

independent units, and they are usually taken to be mass (M), length (L) and time (T). Like the ratios of force, mass and velocity, the ratios of the fundamental units must also have a constant value at corresponding points throughout the systems (for example $F^* = M^*L^*T^{*-2}$ and $V^* = L^*T^{*-1}$).

2. The problem

The problem with the models described by Del Ventisetti et al. (2006, Table 2 and Appendix A) is that there is no consistent set of dimensionless ratios that have constant values at corresponding points relating the model to nature. As a result, there can be no claim that the models have anything to do with natural deformation.

In Appendix A, for example, the authors use length (l^*), density (ρ^*) and gravity (g^*) to determine the stress ratio ($\sigma^* = l^*\rho^*g^*$). Once those three parameters have been selected to define the scaling parameters, the fundamental dimensions M^*, L^* and T^* have also been defined. But in Eqs. (6) and (7) of Appendix A, the authors seem to use velocity in the model (not gravity), the thickness of what appears to be an undefined shear zone in the model (not L^*), and the strain rate in nature (not g^* and l^*) to find T^*. Actually, I found it difficult to tell what is happening in Eqs. (6) and (7) because the explanation is incomplete.[1]

This problem of inconsistent scaling ratios can best be shown by calculating the three fundamental ratios of mass, length and time from the author's five dimensionless ratios of length (l^*), viscosity (η^*), density (ρ^*), gravity (g^*), and time (t^*) determined from measurements of those parameters in the model and estimates of those same parameters in nature; these are reproduced here in Table 1 from Table 2 and Appendix A in Del Ventisetti et al. (2006).

For the model to legitimately represent the natural process, the dimensionless ratios of the fundamental units (M, L, T) must all have the same value for each of the parameters that describe the physical system. In other words $\rho^* = 0.5 = M^*L^{*-3}$; $\eta^* = 2E-15 = M^*L^{*-1}T^{*-1}$; $\underline{v}^* = 3.42E+04 = L^*/T^*$; etc.

Since there are three fundamental, independent dimensionless ratios, it takes three of the five measured dimensionless parameters to calculate them. So, any three combinations of the five must give the same values of the fundamental ratios if the model represents nature.

Tables 2–5 show four combinations of the five measured dimensionless ratios. The values of the five measured dimensionless parameters used in the experiments do not have consistent fundamental ratios as shown in Tables 2–5. The time ratio T^*, varies by 14 orders of magnitude, M^* by 42, and L^* by 15 depending on which three of the five parameters are used.

Table 5 also illustrates what Hubbert pointed out in 1937: if gravity cannot be neglected in the natural process, it is

Table 2
M^*, L^*, T^* model ratios using l^*, ρ^* and t^* (length, density and time)

$T^* = t^*$	2.77E − 10	From time ratio
$M^* = \rho^* l^{*3}$	5.00E − 16	From density and length ratios
$L^* = l^*$	1.00E − 05	From length ratio

Table 3
M^*, L^*, T^* model ratios using l^*, ρ^* and η^* (length, density and viscosity)

$T^* = \rho^* l^{*2}/\eta^*$	2.41E + 04	From viscosity, mass and length ratios
$M^* = \rho^* l^{*3}$	5.00E − 16	From density and length ratios
$L^* = l^*$	1.00E − 05	From length ratio

Table 4
M^*, L^*, T^* model ratios using t^*, ρ^* and η^* (time, density and viscosity)

$T^* = t^*$	2.77E − 10	From time ratio
$M^* = \rho^*(t^*\eta^*/\rho^*)^{1.5}$	5.83E − 37	From density and viscosity ratios
$L^* = (t^*\eta^*/\rho^*)^{0.5}$	1.05E − 12	From viscosity, time and density ratios

Table 5
M^*, L^*, T^* model ratios using t^*, ρ^* and g^* (time, density and gravity)

$T^* = t^*$	2.77E − 10	From time ratio
$M^* = \rho^*(g^*t^{*2})^3$	2.25E − 58	From density, gravity and time ratios
$L^* = g^*t^{*2}$	7.67E − 20	From gravity and time ratios

[1] For example one variable (γ_d) is described as shear strain when it should be shear strain rate; another (H_d) is not defined at all and is presumably the width of a shear zone; and, without further definition, velocity (v) has little meaning in a deforming Newtonian fluid.

impossible to find real materials that can be used in a table top model that reproduces that process. Several researchers have solved this problem by using a centrifuge to run models.

3. Conclusion

As a general rule for materials that are time dependent, gravity cannot be neglected if there is a surface (topographic) gradient or any other horizontal density gradient in the natural system. How small those gradients need to be in order to be neglected would depend on the system being modeled. In the basin inversion system modeled by Del Ventisetti et al. (2006), viscous materials with horizontal density gradients seem to be a central feature in the deformation, so gravity must be included in the scaling parameters. The authors do include gravity in calculating the stress ratio (σ*), but fail to use it when calculating time and rate ratios. So, either their time ratio or the stress ratio is invalid.

Models using sand and clay are frequently used because the deformation patterns often look similar to natural systems. I suspect the reason for that similarity is that three important variables (angles, strain and coefficients of friction) are dimensionless. Dimensionless variables are probably very forgiving and models that are not scaled may look realistic for that reason. However, attempts to relate dimensional variables in a model to natural systems with horizontal density gradients are surely invalid without careful scaling that includes gravity.

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